SCIENCE NEWS FEATURE • NUMBER THEORY & ARTIFICIAL INTELLIGENCE

Beyond the 17-Million-Line Machine Proof: How a Seven-Page Coordinate Sieve Offers a Human-Scale Approach to Prime Numbers

As computer-verified theorem proving pushes analytic number theory into multi-million-line codebases, a compact three-coordinate counting architecture illustrates an alternative path grounded in transparent modular arithmetic.

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THE HAGUE, Netherlands — November 24, 2026 — Mathematics entered uncharted territory on November 20, 2026, when Anthropic mathematician Levent Alpöge announced a formal proof of the 167-year-old Riemann Hypothesis—one of the Clay Mathematics Institute's seven Millennium Prize Problems—developed over six weeks in collaboration with the artificial intelligence system Claude.

Rather than a traditional journal paper, Alpöge's proof arrived as a 61.5-gigabyte archive of 175,187 modules totaling roughly 17 million lines of code in Lean 4, a computer-assisted theorem prover. While the Lean Formalization Research Organization, the Mathlib Initiative, and independent mathematicians at MIT and the University of Waterloo confirmed within 48 hours that the codebase compiles cleanly and matches the formal statement of the Riemann Hypothesis, the sheer scale of the machine-generated derivation—jokingly estimated by Alpöge as exceeding 250,000 pages if printed in raw prose—has sparked an urgent debate across the mathematical community: *If an AI proves a foundational theorem in millions of lines of code that no human can read end-to-end, how do we preserve human mathematical intuition?*

A new open-science manuscript by Patrick Merrick, an MSc alumnus of the University of Bath based in The Hague, illustrates the opposite end of that spectrum: a compact, seven-page deterministic architecture for locating prime numbers that relies entirely on transparent modular arithmetic and human-auditable counting rules.

Two Different Questions About the Primes

Although both efforts center on the distribution of prime numbers, they address fundamentally different mathematical problems:

- **Asymptotic Distribution (The Riemann Hypothesis):** Proving that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\mathrm{Re}(s) = 1/2$ resolves the global asymptotic error bound for how densely primes are distributed as numbers approach infinity. Even when verified in Lean, translating Alpöge and Claude's 175,187-file formalization into human-readable conceptual mathematics is expected to take years.

- **Exact Local Construction (The Three-Coordinate Counting Sieve):** In his manuscript, *Predicting Prime: A Hierarchical Three-Coordinate Model and Deterministic Counting-Sieve Architecture for Prime Number Distribution*, Merrick tackles the practical, deterministic identification of exact prime locations across finite integer ranges without relying on trial division or black-box computation.

Dimension	AI-Assisted RH Formalization (Alpöge & Claude, 2026)	Three-Coordinate Counting Sieve (Merrick, 2026)
Mathematical Scope	Global asymptotic distribution of prime numbers as $x \to \infty$ via zeros of $\zeta(s)$	Exact deterministic construction and location of all primes up to any finite bound $N_{\max}$
Format & Scale	175,187 Lean 4 modules (~17 million lines of formal code; >250,000 pages in raw printout)	7-page manuscript with deterministic grid coordinates and spreadsheet-auditable sieve rules
Verification Mode	Machine-checked formal type verification (Lean FRO, Mathlib Initiative)	Direct arithmetic verification across 100 Major Groups (N = 1 to 7,000; 900 primes, 100% accuracy)

How the Three-Coordinate Model Works

Merrick's framework maps every positive integer N onto a three-tier coordinate grid (G, M, P) governed by the linear equation:

$$N(G, M, P) = 70(G - 1) + 10(M - 1) + P$$

Here, $G \geq 1$ indexes consecutive 70-integer **Major Groups**, $M \in \{1, \ldots, 7\}$ indexes seven 10-integer **Minor Groups** (labelled descendingly from Q to K), and $P \in \{1, \ldots, 10\}$ denotes the slot within each decade.

By anchoring the Major Group length to $70 = 2 \times 5 \times 7$ and analyzing three-group super-blocks of the fourth primorial $210 = 2 \times 3 \times 5 \times 7$, the grid acts as a rigid modular filter:

1. **Positional Elimination (77.14% Reduction):** Across every 210-integer cycle, positional periodicity modulo 2, 3, 5, and 7 automatically eliminates 162 composite numbers, leaving exactly $\varphi(210) = 48$ candidate coordinates without performing a single division.
2. **Deterministic Forward Step-Counting:** To filter out composites formed by larger prime factors ($p \geq 11$), the model applies a forward counting sieve along arithmetic progressions $S_p = \{p^2 + kp \mid k \geq 0\}$ for confirmed primes $11 \leq p \leq \sqrt{N_{\max}}$.

In benchmark verification across a complete 100-Major-Group Registry (N = 1 to 7,000, spanning 700 decades), the architecture isolated all 900 prime numbers with 100% accuracy—zero false positives and zero false negatives—requiring only the 19 sieving primes from 11 to 83 (a rule set that remains exact up to N = 7,920, just below 89^2).

Complementary Paradigms in Modern Number Theory

The juxtaposition of a 17-million-line AI-assisted formal proof and a seven-page deterministic coordinate sieve underscores a growing duality in twenty-first-century mathematics. Machine-assisted formal verification pushes the frontier of deep analytic conjectures beyond human scale, while compact, structured representations—such as modular wheel coordinate systems—keep computational number theory transparent, verifiable by hand or spreadsheet, and accessible to researchers and educators alike.

References & Data Availability

Merrick, P. (2026). *Predicting Prime: A Hierarchical Three-Coordinate Model and Deterministic Counting-Sieve Architecture for Prime Number Distribution*. Prepared for Zenodo Open Science Repository & ORCID Indexing.

Alpöge, L. (2026). *Remark* & Lean 4 Formalization Archive of the Riemann Hypothesis and Generalized Riemann Hypothesis (verified by Lean FRO and Mathlib Initiative, November 21, 2026).

Predicting Prime

A Hierarchical Three-Coordinate Model and Deterministic Counting-Sieve Architecture for Prime Number Distribution

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Abstract

This manuscript presents a deterministic, three-level coordinate framework and counting-sieve methodology for locating and predicting prime numbers across the positive integers. Beginning from an empirical decade-level grid of consecutive ten-integer intervals (labelled alphabetically from Q to A), the model establishes a hierarchical coordinate system governed by the linear mapping $N(G, M, P) = 70(G-1)+10(M-1)+P$, where $G \in \mathbb{Z}_{\geq 1}$ denotes the 70-integer Major Group, $M \in \{1, \ldots, 7\}$ denotes the 10-integer Minor Group (Q, P, O, N, M, L, K), and $P \in \{1, \ldots, 10\}$ denotes the individual decade position. By anchoring the Major Group period to $70 = 2 \times 5 \times 7$ and analyzing three-major-group super-blocks of $210 = 2 \times 3 \times 5 \times 7$, the model isolates a rigid, repeating modular counting structure that eliminates 77.14% of all composite integers (162 of every 210 numbers) purely through positional periodicity. To resolve the remaining 48 wheel-surviving candidates per 210-integer cycle without trial division, a deterministic forward counting-sieve is integrated across confirmed primes $11 \leq p \leq \sqrt{N_{\max}}$, eliminating step-landings $\{p^2 + kp \mid k \in \mathbb{Z}_{\geq 0}\}$. Computational verification across the complete 100-Major-Group Registry ($N = 1$ to $7,000$; 700 minor groups) demonstrates 100% predictive accuracy, isolating all 900 prime numbers with zero false positives and zero false negatives, and remaining exact up to $N = 7,920$ ($89^2 - 1$).

Keywords: Prime number distribution; Three-coordinate integer mapping; Wheel factorization; Counting sieve; Sieve of Eratosthenes; Modular periodicity; 100-Major-Group Registry.

1 Introduction and Background

The distribution of prime numbers among the positive integers exhibits a dual character: globally irregular in exact local spacing, yet strictly constrained by modular arithmetic. Traditional primality testing relies on trial division or algebraic criteria applied to individual integers in isolation. In contrast, positional grid models seek to reveal structural regularities by partitioning the number line into uniform intervals and observing where prime positions occur.

This investigation originated from a positional grid constructed over consecutive groups of ten integers, wherein rows were assigned descending alphabetic labels ($Q, P, O, N, M, L, K, J, I, H, G, F, E,$ and columns indexed individual positions 1 through 10. Marking prime locations with X and composite locations with O across the first 17 decades (1 to 170) prompted two foundational research questions:

1. Does the initial 17-row positional table accurately capture all prime positions within 1 to 170, and what structural rules govern valid columns when extended to $1,000$ (100 decades)?

2. Can a hierarchical grouping of consecutive decades reveal larger repeating cycles, and can a structured counting process over these coordinates deterministically predict exact prime numbers without performing trial division?

Through iterative empirical verification, expansion to 100 Major Groups ($N = 1$ to $7,000$), and formalization via modular wheel mechanics, this work demonstrates that combining a three-coordinate integer grid with a forward step-counting sieve yields an exact, deterministic prime-generating architecture.

2 Verification of the Initial Decade Model (1–170)

In the foundational grid, each row represents a decade of ten consecutive positive integers ($Q = 1$–10, $P = 11$–20, $O = 21$–30, continuing sequentially to $A = 161$–170). Mathematical audit of the initial 17-row table identified discrepancies between visually marked X positions and true primality, establishing the exact baseline shown in Table 1.

Table 1: Mathematically verified prime positions and primes across the initial 17 decades (1–170).

Row	Range	Positions	Primes	Row	Range	Positions	Primes
Q	1–10	2, 3, 5, 7	2, 3, 5, 7	H	91–100	7	97
P	11–20	1, 3, 7, 9	11, 13, 17, 19	G	101–110	1, 3, 7, 9	101, 103, 107, 109
O	21–30	3, 9	23, 29	F	111–120	3	113
N	31–40	1, 7	31, 37	E	121–130	7	127
M	41–50	1, 3, 7	41, 43, 47	D	131–140	1, 7, 9	131, 137, 139
L	51–60	3, 9	53, 59	C	141–150	9	149
K	61–70	1, 7	61, 67	B	151–160	1, 7	151, 157
J	71–80	1, 3, 9	71, 73, 79	A	161–170	3, 7	163, 167
I	81–90	3, 9	83, 89				

From elementary divisibility, any integer ending in $0, 2, 4, 6,$ or 8 is divisible by 2, and any integer ending in 5 is divisible by 5. Consequently, beyond the initial row Q (which contains the base primes 2 and 5), prime numbers in every subsequent decade can only occupy positions $P \in \{1, 3, 7, 9\}$. Extending the grid to $1,000$ requires exactly 100 rows of ten, where candidate positions remain permanently restricted to $\{1, 3, 7, 9\}$ while actual occupancy varies due to higher composite factors ($21 = 3 \times 7$, $27 = 3^3$, $33 = 3 \times 11$, $39 = 3 \times 13$, etc.).

3 Hierarchical Grouping and Three-Coordinate Architecture

To investigate macro-level organization beyond single decades, the number line was structured into a three-tier coordinate hierarchy:

- **Level 1 — Major Group (G):** Consecutive blocks of 70 integers (7 decades), indexed numerically as $G = 1, 2, 3, \ldots$
- **Level 2 — Minor Group (M):** Seven consecutive decades within each Major Group, preserving the original descending alphabetic identifiers Q, P, O, N, M, L, K corresponding to indices $M = 1, 2, 3, 4, 5, 6, 7$.
- **Level 3 — Individual Position (P):** Ten sequential slots within each Minor Group, indexed $P = 1, 2, \ldots, 10$.

3.1 The Coordinate Mapping Equation

Every positive integer $N \in \mathbb{Z}_{\geq 1}$ is uniquely addressed by its ordered triple (G, M, P) through the exact linear mapping:

$$N(G, M, P) = 70(G - 1) + 10(M - 1) + P \tag{1}$$

For example, the coordinate triple $(G = 177, M = 1\,[Q], P = 3)$ evaluates to:

$$N(177, 1, 3) = 70(177 - 1) + 10(1 - 1) + 3 = 12,320 + 0 + 3 = 12,323$$

which identifies a verified prime number directly from its hierarchical coordinates.

3.2 First Ten Major Groups (1–700) and Non-Periodicity of Exact Signatures

Applying this hierarchy across the first ten Major Groups ($N = 1$ to 700) yields the prime positional signatures summarized in Table 2. An empty bracket [] denotes a minor group containing zero primes.

Table 2: Prime positional arrangements across Major Groups 1 through 10 ($N = 1$–700).

Major (G)	Range	Seven Minor-Group Arrangements (Q, P, O, N, M, L, K)	Total Primes
1	1–70	$[2,3,5,7]$ $[1,3,7,9]$ $[3,9]$ $[1,7]$ $[1,3,7]$ $[3,9]$ $[1,7]$	19
2	71–140	$[1,3,9]$ $[3,9]$ $[7]$ $[1,3,7,9]$ $[3]$ $[7]$ $[1,7,9]$	15
3	141–210	$[9]$ $[1,7]$ $[3,7]$ $[3,9]$ $[1]$ $[1,3,7,9]$ []	12
4	211–280	$[1]$ $[3,7,9]$ $[3,9]$ $[1]$ $[1,7]$ $[3,9]$ $[1,7]$	13
5	281–350	$[1,3]$ $[3]$ $[7]$ $[1,3,7]$ [] $[1,7]$ $[7,9]$	11
6	351–420	$[3,9]$ $[7]$ $[3,9]$ $[3,9]$ $[7]$ $[1,9]$ $[9]$	11
7	421–490	$[1]$ $[1,3,9]$ $[3,9]$ $[7]$ $[1,3,7]$ $[9]$ $[7]$	12
8	491–560	$[1,9]$ $[3,9]$ [] $[1,3]$ [] $[1,7]$ $[7]$	9
9	561–630	$[3,9]$ $[1,7]$ $[7]$ $[3,9]$ $[1,7]$ $[3,7,9]$ []	12
10	631–700	$[1]$ $[1,3,7]$ $[3,9]$ $[1]$ $[3,7]$ $[3]$ $[1]$	11

Examination of Major Groups 1 through 10 demonstrates that while prime density exhibits a general downward trend (from 19 primes in $G = 1$ to 11 primes in $G = 10$, with local fluctuations such as $G = 8$ dropping to 9 primes before rebounding to 12 in $G = 9$), no two seven-minor-group signatures are identical within 1–700.

3.3 Exact Signature Recurrence Analysis

Extending the computational search beyond 700 to locate the first exact repetition of an entire seven-minor-group signature identifies the common arrangement:

$$Q : [3,9], \quad P : [], \quad O : [3,7], \quad N : [], \quad M : [], \quad L : [3,7,9], \quad K : []$$

which contains exactly 7 primes per Major Group. This identical signature occurs at:

- **First Occurrence:** Major Group $G = 177$ ($12,321$–$12,390$), primes $\{12323, 12329, 12343, 12347, 12373, 12377,$
- **Second Occurrence:** Major Group $G = 522$ ($36,471$–$36,540$), separation of 345 Major Groups ($24,150$ integers).
- **Third Occurrence:** Major Group $G = 773,433$ ($54,140,241$–$54,140,310$), separation of $772,911$ Major Groups ($54,103,770$ integers).

Because the intervals between consecutive recurrences (345 vs. $772,911$ Major Groups) are non-uniform, exact Major Group signatures recur aperiodically. True predictability arises not from waiting for full signature repetition, but from the modular wheel structure embedded in 70 and 210.

4 Modular Wheel Mechanics and Pure Counting Periodicity

The selection of 70 integers per Major Group carries a fundamental number-theoretic property:

$$70 = 2 \times 5 \times 7, \qquad 210 = 3 \times 70 = 2 \times 3 \times 5 \times 7 \tag{2}$$

Because 70 is the primorial $5\# \times 7 = 70$ and 210 is the fourth primorial $7\# = 210$, stepping through (G, M, P) establishes an invariant counting filter for the first four prime numbers ($2, 3, 5,$ and 7):

1. **Divisibility by 2 and 5 (Period = 1 Minor Group / 10 integers):** In every Minor Group M, multiples of 2 and 5 occupy positions $\{2, 4, 5, 6, 8, 10\}$, leaving strictly 4 candidate positions $\{1, 3, 7, 9\}$ per decade (28 candidates per Major Group).
2. **Divisibility by 7 (Period = 1 Major Group / 70 integers):** Because $70 \equiv 0 \pmod 7$, multiples of 7 fall on the *exact same* (M, P) coordinates in every Major Group G:

   $$Q : [7], \quad P : [4], \quad O : [1, 8], \quad N : [5], \quad M : [2, 9], \quad L : [6], \quad K : [3, 10]$$

   Removing positions divisible by 7 from the 28 candidates leaves exactly 24 survivors per Major Group.
3. **Divisibility by 3 (Period = 3 Major Groups / 210 integers):** Because $70 \equiv 1 \pmod 3$, incrementing $G \to G + 1$ shifts multiples of 3 by one position modulo 3, cycling through three residue classes ($G \bmod 3 \in \{1, 2, 0\}$):
   - $G \equiv 1 \pmod 3$: multiples of 3 occupy $\{3, 6, 9\}$ in Q, N, K; $\{2, 5, 8\}$ in P, M; and $\{1, 4, 7\}$ in O, L.
   - $G \equiv 2 \pmod 3$: multiples of 3 occupy $\{2, 5, 8\}$ in Q, N, K; $\{1, 4, 7\}$ in P, M; and $\{3, 6, 9\}$ in O, L.
   - $G \equiv 0 \pmod 3$: multiples of 3 occupy $\{1, 4, 7\}$ in Q, N, K; $\{3, 6, 9\}$ in P, M; and $\{2, 5, 8\}$ in O, L.

Across any complete three-Major-Group super-block of 210 integers, pure positional counting for $2, 3, 5,$ and 7 eliminates exactly 162 composite numbers (77.14%) and leaves exactly $\varphi(210) = (2-1)(3-1)(5-1)(7-1) = 48$ candidate coordinates.

5 The 100-Major-Group Registry (1–7,000) and Counting Sieve

While the 210-wheel counting structure eliminates all multiples of $2, 3, 5,$ and 7, composite integers formed exclusively by primes $p \geq 11$ (such as $121 = 11^2$, $143 = 11 \times 13$, $169 = 13^2$, or $221 = 13 \times 17$) remain among the 48 candidates per 210 numbers.

5.1 Resolution via Forward Step-Counting

Rather than testing surviving coordinates by trial division, primality is resolved purely by counting forward along confirmed prime intervals. Once a wheel-surviving coordinate is con-

firmed as a prime $p \geq 11$, its composite multiples begin at p^2 (since any smaller multiple mp with $m < p$ has already been eliminated by a smaller prime factor) and recur at exact step intervals of $+p$:

$$S_p = \{p^2 + kp \mid k \in \mathbb{Z}_{\geq 0}\} \tag{3}$$

For the complete 100-Major-Group Registry ($N_{\text{max}} = 7,000$), the upper bound for sieving primes is $\lfloor \sqrt{7,000} \rfloor = 83$, requiring only the 19 primes:

$$\mathbb{P}_{11\text{–}83} = \{11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83\}$$

Every wheel-surviving integer $N(G, M, P) \leq 7,000$ that is not landed upon by any sequence S_p ($p \in \mathbb{P}_{11\text{–}83}$) is deterministically guaranteed to be prime. Furthermore, because the next prime after 83 is 89 and $89^2 = 7,921$, this exact 19-prime counting sieve remains 100% accurate up to $N = 7,920$ (Major Group 113, Minor Group O, Position 10).

5.2 Empirical Results of the 100-Major-Group Registry

Executing the three-coordinate registry and counting sieve across all 100 Major Groups (700 Minor Groups; $7,000$ integers) yields the verified aggregate metrics reported in Table 3 and the minor-group distribution in Table 4.

Table 3: Summary statistics and verification audit of the 100-Major-Group Registry ($N = 1$–$7,000$).

Metric	Value	Notes / Coordinates
Total Integers Evaluated	7,000	Major Groups $G = 1$ to 100 (700 Minor Groups)
210-Wheel Surviving Candidates	1,603	Includes base primes $\{2, 3, 5, 7\}$ + $1,599$ coprime to 210
Composites Eliminated by $p \in [11, 83]$	703	100% captured by step-landings $p^2 + kp$
Verified Prime Numbers ($\pi(7000)$)	**900**	**100.0% accuracy (0 false positives, 0 false negatives)**
Mean Primes per Major Group	9.00	Mean of 1.29 primes per Minor Group
Maximum Primes in a Major Group	19	Major Group 1 (1–70)
Minimum Primes in a Major Group	6	Major Groups $44, 50, 56, 60, 62, 69, 73, 77, 81, 84, 87, 90, 97$
Empty Minor Groups (0 Primes)	183	26.14% of all 700 decades contain zero primes

Table 4: Distribution of verified primes and individual positions across Minor Group identifiers Q–K ($G = 1$–100).

Minor ID	Index (M)	Total Primes	Pos 1	Pos 3	Pos 7	Pos 9	Pos 2 & 5	Empty Decades
Q	1	130	31	32	34	31	2	24
P	2	129	32	34	31	32	0	27
O	3	131	32	33	33	33	0	24
N	4	126	31	32	32	31	0	25
M	5	129	33	33	31	32	0	27
L	6	126	32	30	32	32	0	31
K	7	129	31	33	33	32	0	25
Total	**1–7**	**900**	**222**	**227**	**226**	**223**	**2**	**183**

As shown in Table 4, prime occurrences are distributed nearly uniformly across the seven Minor Group identifiers Q through K (ranging between 126 and 131 primes per identifier) and across the four valid residue positions $\{1, 3, 7, 9\}$ (222 to 227 primes per position), consistent with Dirichlet's theorem on primes in arithmetic progressions. Representative excerpts from the 100-Major-Group Summary and Detailed Registry are presented in Tables 5 and 6.

Table 5: Selected Major Group summary records from the 100-Major-Group Registry ($G = 1$–5 and $G = 96$–100).

Major (G)	Range	Primes	Empty Decades	Pos 1	Pos 3	Pos 7	Pos 9
1	1–70	19	0	4	5	5	3
2	71–140	15	0	3	3	4	5
3	141–210	12	1	3	3	3	3
4	211–280	13	0	4	3	3	3
5	281–350	11	1	3	3	4	1
96	6651–6720	11	1	3	2	3	3
97	6721–6790	6	2	2	1	1	2
98	6791–6860	9	1	2	3	2	2
99	6861–6930	8	1	2	3	2	1
100	6931–7000	10	1	3	3	1	3

Table 6: Detailed elimination structure for Major Group 1 ($N = 1$–70).

G	Minor	Range	Possible	Div by 3	Div by 7	Verified Pos	Verified Primes
1	Q	1–10	2, 3, 5, 7	3, 6, 9	7	2, 3, 5, 7	2, 3, 5, 7
1	P	11–20	1, 3, 7, 9	2, 5, 8	4	1, 3, 7, 9	11, 13, 17, 19
1	O	21–30	1, 3, 7, 9	1, 4, 7	1, 8	3, 9	23, 29
1	N	31–40	1, 3, 7, 9	3, 6, 9	5	1, 7	31, 37
1	M	41–50	1, 3, 7, 9	2, 5, 8	2, 9	1, 3, 7	41, 43, 47
1	L	51–60	1, 3, 7, 9	1, 4, 7	6	3, 9	53, 59
1	K	61–70	1, 3, 7, 9	3, 6, 9	3, 10	1, 7	61, 67

6 Formal Mathematical Formulation

Combining the three-coordinate mapping function (Equation 1), the 210-wheel coprimality filter, and the forward step-counting sieve produces a complete set-theoretic and indicator formulation for predicting prime numbers up to any chosen upper bound $N_{\max} = 70G_{\max}$.

6.1 Prime Set Formulation

Let $\mathbb{P}$ denote the set of all prime numbers up to $N_{\max} = 70G_{\max}$, where $G \in \{1, \ldots, G_{\max}\}$, $M \in \{1, \ldots, 7\}$, and $P \in \{1, \ldots, 10\}$. Then:

$$\mathbb{P}_{\leq N_{\max}} = \{2, 3, 5, 7\} \cup \left\{ N(G, M, P) \;\middle|\; \gcd\big(N(G, M, P),\, 210\big) = 1 \;\;\wedge\;\; \forall\, p \in \mathbb{P}_{11 \leq p \leq \sqrt{N_{\max}}},\; N(G, M, P) \notin \{p^2 + k \} \right. \tag{4}$$

Here:

- $\{2, 3, 5, 7\}$ initializes the four base primes defining the 210-wheel ($210 = 2 \times 3 \times 5 \times 7$).
- $\gcd\big(N(G, M, P),\, 210\big) = 1$ enforces the repeating $(G \bmod 3, M, P)$ wheel filter that eliminates 1 and all multiples of $2, 3, 5$, and 7.
- $N(G, M, P) \notin \{p^2 + kp \mid k \in \mathbb{Z}_{\geq 0}\}$ removes every composite integer reached by counting forward in steps of p starting from p^2 for all confirmed primes $11 \leq p \leq \sqrt{N_{\max}}$.

6.2 Primality Indicator Function

For any coordinate triple (G, M, P) yielding $N(G, M, P) > 7$, the binary primality indicator function $I(G, M, P) \in \{0, 1\}$ is defined as:

$$I(G,M,P) = \begin{cases} 1, & \text{if } \gcd\big(N(G,M,P),\,210\big) = 1 \text{ and } \displaystyle\prod_{\substack{p\in\mathbb{P}\\ 11\le p\le\sqrt{N}}} \big(N(G,M,P) \bmod p\big) \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $I(G, M, P) = 1$ if and only if $N(G, M, P)$ is prime, and $I(G, M, P) = 0$ if and only if $N(G, M, P)$ is composite.

7 Discussion and Conclusions

This investigation resolves the relationship between positional decade grids and deterministic prime prediction. While consecutive 70-integer Major Groups do not repeat exact seven-decade prime signatures at periodic intervals, embedding the 70-integer Major Group inside a three-Major-Group (210-integer) wheel transforms the (G, M, P) coordinate system into an exact modular filter. Pure positional counting across (G mod $3, M, P$) eliminates 77.14% of all integers ($2, 3, 5$, and 7), and pairing the remaining 48 candidates per 210-integer block with forward step-counting from p^2 in increments of p for $11 \le p \le \sqrt{N_{\text{max}}}$ deterministically isolates every prime without trial division.

Across the verified 100-Major-Group Registry ($N = 1$ to $7,000$), the framework identifies all 900 primes with 100% precision and scales cleanly to arbitrary upper bounds by appending a single counting column S_p whenever N_{max} reaches p^2.

Data Availability Statement

The complete computational dataset supporting this manuscript—comprising the *Overview & Analytics*, *Major Group Summary* ($G = 1$–100), *Detailed Registry* (700 Minor Groups), and live formula-driven *Counting Sieve (*11–83*)*—is compiled in the accompanying workbook `Prime_Groups_100_M` for open archival alongside this publication.